On the input energy for state reachability of linear systems with packet losses

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Introduction

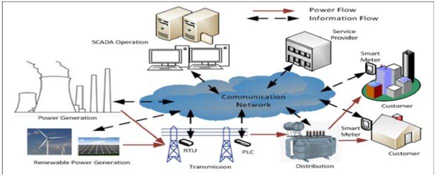
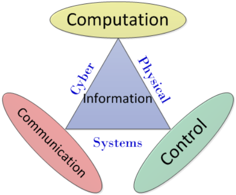
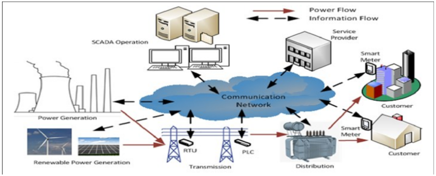


Figure : Cyber Physical Systems





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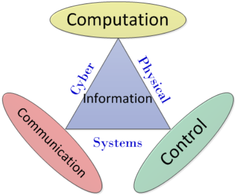

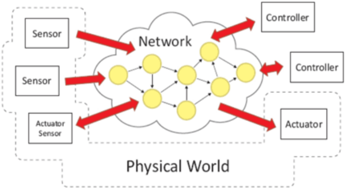

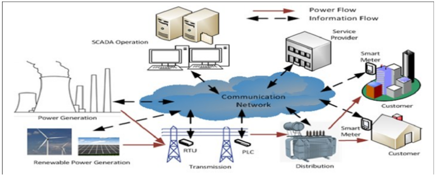

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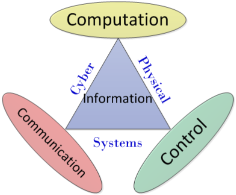

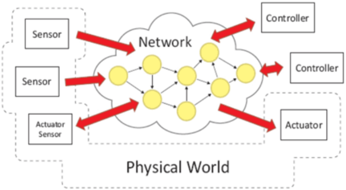

- Packet loss in wireless communication: a common non-ideality.

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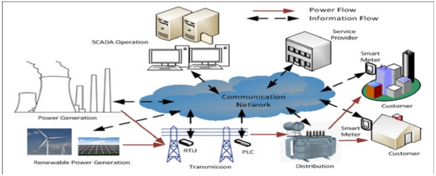

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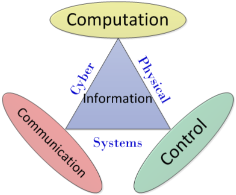

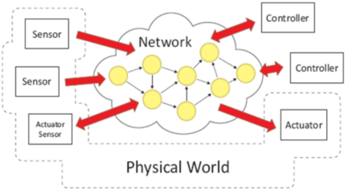

- Packet loss in wireless communication: a common non-ideality.
- Greatly influences controllability, observability, required control energy etc.
- [Jungers, Kundu and Heemels]: controllability and observability are decidable.

Model for discrete linear systems with packet dropouts

- Discrete linear systems subject to data losses :
  $x(t+1) = Ax(t) + B\sigma(t)u(t)$ where

hybrid systems [Jungers, Kundu and Heemels]

$$x(t+1) = \begin{cases} Ax(t) + Bu(t), & \text{if } \sigma(t) = 1 \\ Ax(t), & \text{if } \sigma(t) = 0. \end{cases} \quad (1)$$

- σ is a signal which models the packet loss.

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- σ is a signal which models the packet loss.
- We say that a signal σ is admissible if it is allowed by some automaton.

Definition ([Jungers, Kundu and Heemels])

We say that the hybrid system is controllable, if for all admissible signals $\sigma : \mathbb{N} \to \{0,1\}$, any initial state $x_0 \in \mathbb{R}^n$ and any final state $x_f \in \mathbb{R}^n$, there is an input signal u such that

$$x_{x_0,\sigma,u(T)} = x_f \text{ for some } T \in N.$$

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We want to study how the energy for reachability is affected when there are packet dropouts.

Controllability Gramian for linear systems

- The Controllability Gramian for discrete linear system

$$W_t(A, B) := \sum_{i=0}^{t} A^i B B^T (A^T)^i. \quad (2)$$

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$$W_t(A,B) \quad := \quad \sum_{i=0}^{t} A^i B B^T (A^T)^i. \tag{2}$$

- If A is stable, then

$$W(A,B) \quad := \quad \sum_{i=0}^{\infty} A^i B B^T (A^T)^i. \tag{3}$$

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- The least input energy required to drive the state from $x(0) = x_0$ to $x(t+1) = x_f$

$$E(x_0, x_f, t) = (x_f - A^t x_0)^T W_t(A,B)^{-1} (x_f - A^t x_0). \tag{4}$$

Metrics on reachability

1. The minimum eigenvalue of the controllability Gramian: $\lambda_{min}(W_t(A, B))$.
2. The trace of the inverse of the controllability Gramian: $\text{tr}(W_t(A, B))^{-1}$.
3. The determinant of the controllability Gramian: $\det(W_t(A, B))$.

- $\lambda_{min}(W_t(A, B)) \leftrightarrow$ maximum energy for reachability on the unit sphere.
- $\text{tr}(W_t(A, B))^{-1} \leftrightarrow$ average energy for reachability on the unit sphere.
- $\det(W_t(A, B)) \leftrightarrow$ volume of the ellipsoid that can be reached with the unit energy input from the origin.

Relevant work

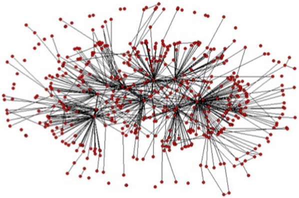

Figure : Complex networks

- [Summers et al.], [Pasqualetti et al.] Problem: optimal actuator placements in complex networks. To choose k number of actuators from a given set to maximize a controllability metric.

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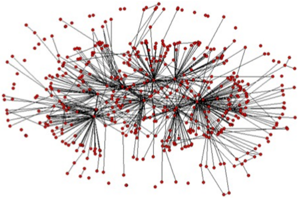

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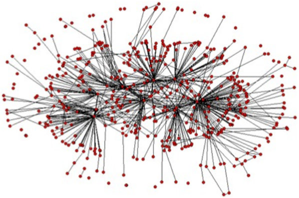

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- [Summers et al.]: Greedy algorithm and the sub-modularity properties of controllability metrics except the minimum eigenvalue of the Controllability Gramian were used to solve this combinatorial problem.
- [Pasqualetti et al.] obtained upper bounds on the minimum eigenvalue in terms of the number of actuators and the number of stable eigenvalues.

Relevant work

- Trade-offs between the input energy and the number of actuators were obtained for large scale networks.
- In specific, lower bounds on the number of actuators was obtained in terms of the fixed input enegy [Pasqualetti et al.].

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- [Olshevsky] obtained upper bounds on the minimum eigenvalue of the Controllability Gramian for linear time invariant systems using tools from potential theory.
- It was shown that if eigenvalues of A are clustered together, it requires more energy to control.
- Our combinatorial problem is different than the one considered in these references and the techniques used therein are not applicable to our case.

Problem statement

- Given a fixed amount of energy or average energy, decide if the reachability problem is feasible for all admissible switching signals; and identify switching signals for which it is infeasible.

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- Provide algorithms to decide this question.
- Quantify the performance of communication networks using the energy required for control.

Automaton and switching signals

Definition

- An automaton is a directed labelled graph $G(V, E)$ with N_V nodes in V and N_E edges in E.
- An edge $(v, w, \sigma) \in E$ carries a label $\sigma \in \{0, 1\}$.
- A sequence $\sigma(0), \sigma(1), \ldots$ is accepted by the graph G if there is a path in G carrying the sequence as the succession of labels on its edges.
- We denote by $\mathcal{L}(\mathcal{A})$ the set of all admissible switching sequences.

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Example ([Jungers, Kundu and Heemels])

- Consider a network where there can be at most 3 consecutive dropouts.

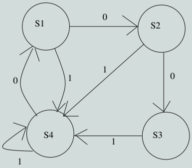

Figure : $\mathcal{A}$

The Controllability Gramian for hybrid systems

- The controllability matrix associated with a signal σ at time t

$$C_{\sigma(t)}(A,B) = \begin{bmatrix} A^t B\sigma(0) & \cdots & AB\sigma(1) & B\sigma(t) \end{bmatrix}. \quad (5)$$

- The controllability Gramian for hybrid systems at time t with respect to the signal σ

$$W_{\sigma(t)} := \sum_{i=0}^{t} \sigma(t-i) A^i B B^T (A^T)^i \quad (6)$$

$$= C_{\sigma(t)}(A,B) C_{\sigma(t)}(A,B)^T. \quad (7)$$

The Controllability Gramian for hybrid systems

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$$C_{\sigma(t)}(A,B) \quad = \quad \left[\begin{array}{cccc} A^t B\sigma(0) & \cdots & AB\sigma(1) & B\sigma(t) \end{array}\right]. \tag{5}$$

- The controllability Gramian for hybrid systems at time t with respect to the signal σ

$$\begin{aligned} W_{\sigma(t)} \quad &:= \quad \sum_{i=0}^{t} \sigma(t-i) A^i B B^T (A^T)^i &(6)\\ &= \quad C_{\sigma(t)}(A,B) C_{\sigma(t)}(A,B)^T. &(7) \end{aligned}$$

- $\bar{u} = C_{\sigma(t)}(A,B)^T (W_{\sigma(t)})^{-1}(x(t+1) - A^t x(0))$ is the minimum energy input which does the required state transfer.
- The minimum input energy required to drive the state from $x(0) = 0$ to $x(t+1) = x_f$ for a switching signal σ is

$$E_\sigma(\{0\},\{x_f\},t) = (x_f)^T W_{\sigma(t)}^{-1}(x_f). \tag{8}$$

Energy and the minimum eigenvalue of the Gramian

- $\max_{x_0=0, x_f \in \mathbb{S}^n} \{E_\sigma(\{0\}, \{x_f\}, t)\} = \lambda_{min}(W_{\sigma(t)})^{-1}$.
- $E(\{0\}, \mathbb{S}^n, t) := \max_{\sigma \in \mathcal{L}(\mathcal{A}), x_0=0, x_f \in \mathbb{S}^n} \{E_\sigma(\{0\}, \{x_f\}, t)\}$.

Energy and the minimum eigenvalue of the Gramian

- $\max_{x_0=0, x_f \in \mathbb{S}^n} \{E_\sigma(\{0\}, \{x_f\}, t)\} = \lambda_{min}(W_{\sigma(t)})^{-1}$.
- $E(\{0\}, \mathbb{S}^n, t) := \max_{\sigma \in \mathcal{L}(\mathcal{A}), x_0=0, x_f \in \mathbb{S}^n} \{E_\sigma(\{0\}, \{x_f\}, t)\}$.
- We are interested in computing the following quantities

$$\lambda(t) := \min_\sigma(\lambda_{min}(W_{\sigma(t)})). \quad (9)$$

$$S_\sigma(t) := \operatorname{argmin}_\sigma(\lambda_{min}(W_{\sigma(t)}). \quad (10)$$

$$\lambda := \lim_{t \to \infty} \lambda(t). \quad (11)$$

- $\lambda(t)^{-1} = E(\{0\}, \mathbb{S}^n, t)$.

Definition

Analogous to the previous definition, we define

$$\bar{S}_\sigma(t) := \text{argmax}_\sigma(\text{tr}(W^{-1}_{\sigma(t)}). \tag{12}$$

Let $\sigma^\dagger \in\in \bar{S}_\sigma(t)$.

$$\delta(t) \quad := \quad \max_\sigma(\text{tr}(W^{-1}_{\sigma(t)})) = \text{tr}(W^{-1}_{\sigma^\dagger(t)}). \tag{13}$$

When A is stable, we define

$$\delta \quad := \quad \lim_{t\to\infty}\delta(t). \tag{14}$$

Computing $\lambda(t)$ and $S_\sigma(t)$ using minimal signals

Definition (partial order)

Given two switching signals σ_1 and σ_2, we say that $\sigma_1 \preceq \sigma_2$ if $\sigma_1(i) = 1$ $(i \in \mathbb{Z})$, then $\sigma_2(i) = 1$ but not conversely. If the converse holds, then $\sigma_1 = \sigma_2$.

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Definition (Minimal signals)

We say that a signal σ is minimal, if there does not exist any other signal $\bar{\sigma}$ $(\bar{\sigma} \neq \sigma)$ allowed by the automaton such that $\bar{\sigma} \preceq \sigma$. We denote by $M_{\sigma(t)}$ the set of all minimal signals defined from 0 to t.

Computing $\lambda(t)$ and $S_\sigma(t)$ using minimal signals

Lemma

Suppose time t is given and $\sigma_1 \preceq \sigma_2$. Then

1. $\lambda_{min}(W_{\sigma_1(t)}) \leq \lambda_{min}(W_{\sigma_2(t)})$.
2. $tr(W^{-1}_{\sigma_1(t)}) \geq tr(W^{-1}_{\sigma_2(t)})$.

Reducing computations

Instead of all signals allowed by the automaton, we need to check the minimal signals to compute $\lambda(t)$ and $S_\sigma(t)$.

How to generate the minimal signals $\rightarrow$ $T-$product lift of graphs

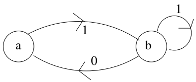
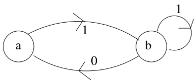


Figure : Automaton

How to generate the minimal signals → $T-$product lift of graphs



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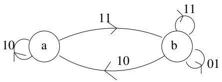

Figure : 2−lift

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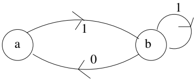

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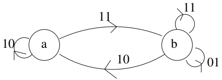

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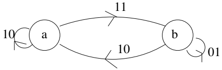

Figure : 2−lift reduction

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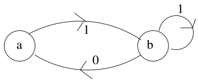

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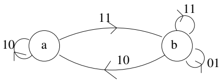

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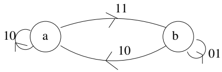

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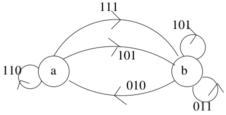

Figure : 3−lift

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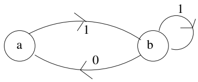

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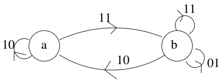

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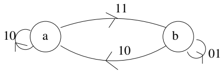

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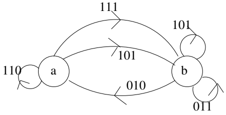

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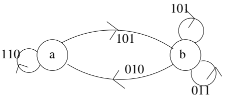

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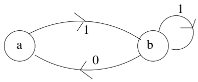

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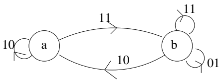

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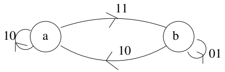

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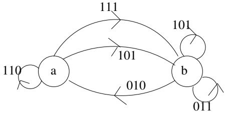

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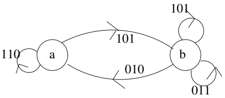

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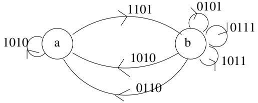
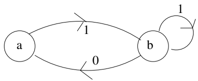


Figure : 4−lift

How to generate the minimal signals → T−product lift of graphs



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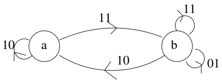

Figure : 2−lift

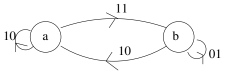

Figure : 2−lift reduction

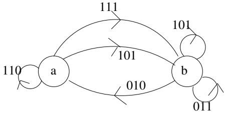

Figure : 3−lift

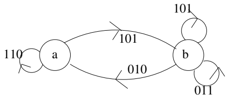

Figure : 3−lift reduction

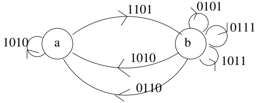

Figure : 4−lift

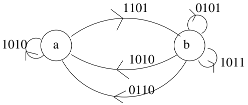

Figure : 4−lift reduction

A Heuristic

- We assume that A is invertible and the system is controllable.

Definition

Let $\mathcal{A}$ be an automaton with m nodes. Let σ be an admissible signal of length m. A signal σ is said to be $m-$minimal if it is a minimal signal with respect to all signals of length m allowed by $\mathcal{A}$.

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Definition (sparse minimal signals)

Let σ be a minimal admissible signal allowed by an automaton $\mathcal{A}$ having m nodes. Partition the time interval from 0 to t into blocks of length m. Then the signal σ is said to be sparse minimal signal if each block in its partition is $m-$minimal. We denote by $Sp_{\sigma(t)}$ the set of all sparse minimal signals from 0 to t.

Definition

We define the approximate of $\lambda(t)$ as follows

$$\lambda_{app}(t) := \min_{\sigma \in Sp_{\sigma(\mathbf{t})}} \{\lambda_{min}(W_{\sigma(t)})\} \tag{15}$$

Example

- Suppose at most 3 successive dropouts are allowed and $t = 7$. Thus $m = 4$.
- m-minimal signals are (1)$\{0, 0, 0, 1\}$, (2)$\{0, 0, 1, 0\}$, (3)$\{0, 1, 0, 0\}$, (4)$\{1, 0, 0, 0\}$.
- The sparse minimal signals are as follows:

$\{0, 0, 0, 1, 0, 0, 0, 1\}$,
$\{0, 0, 0, 1, 0, 0, 1, 0\}$,
$\{0, 0, 0, 1, 0, 1, 0, 0\}$,
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Examples: the number of signals vs time

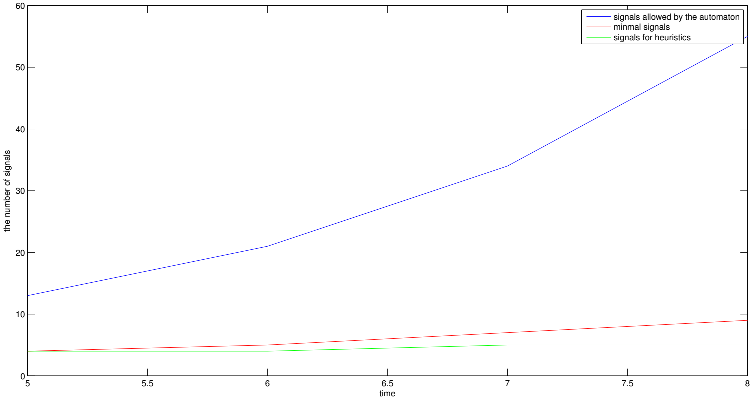

Figure : Automaton: no more than one consecutive dropouts

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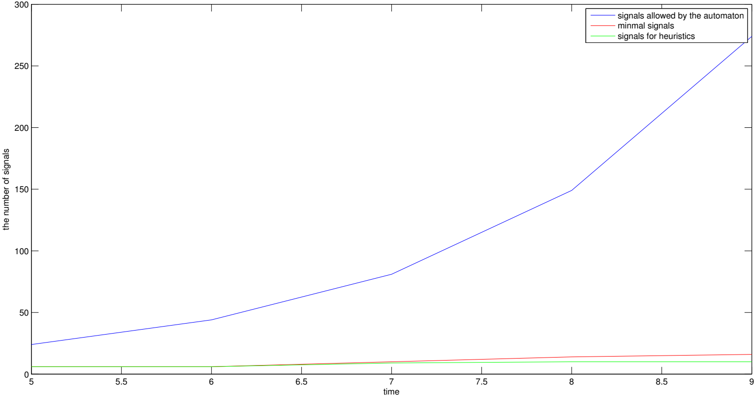
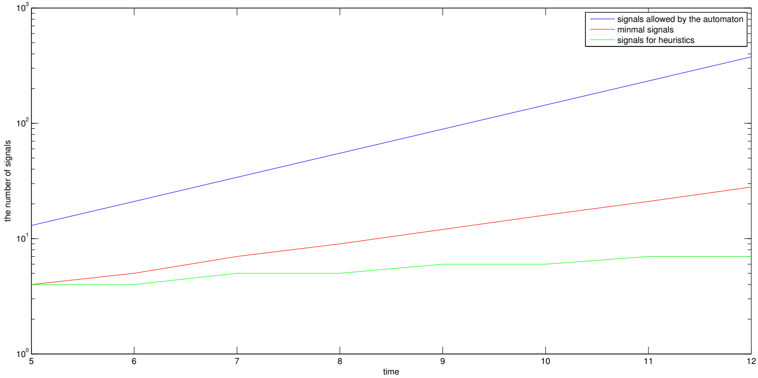


Figure : Automaton: no more than two consecutive dropouts

Examples: the number of signals vs time (logarithmic plot)



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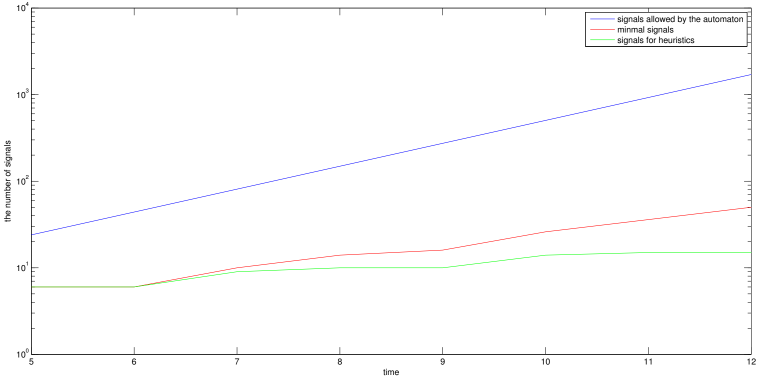

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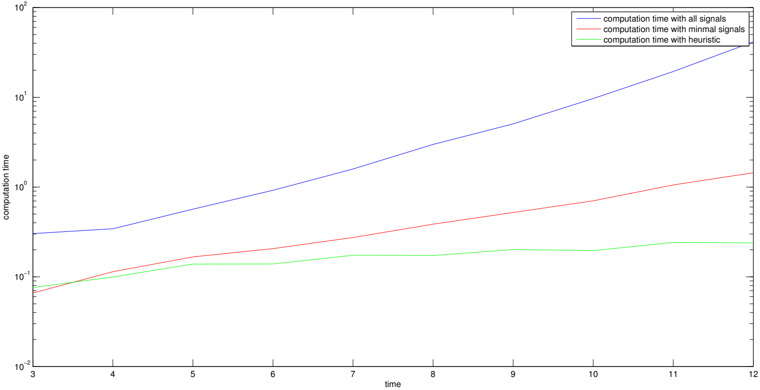

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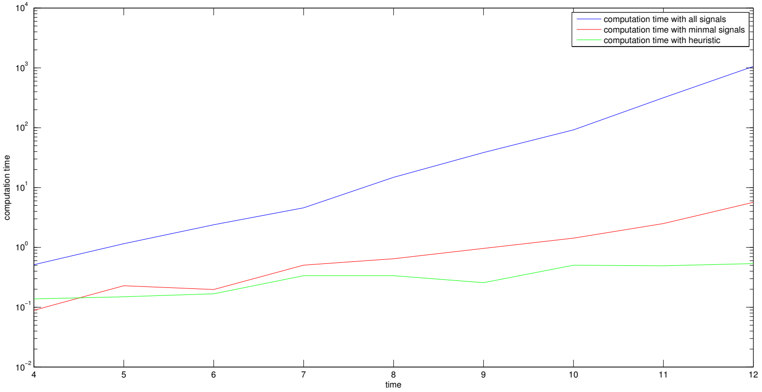

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Evaluation with heuristic

Table : performance of the heuristic for $T = 12$ (relative percentage error)

(No. of dropouts, dimension, no. of inputs)	no. of samples	zero error	error $\leq$ 20%	21% $\leq$ error $\leq$ 60%	61% $\leq$ error $\leq$ 90%	error $>$ 90%
(1,5,3)	2903	2776	2885	13	4	1
(1,10,4)	2822	2750	2810	10	2	0
(2,5,2)	985	893	938	13	5	29
(2,10,5)	2260	2201	2248	6	0	6
(3,5,3)	2320	2320	2320	0	0	0
(3,8,3)	2878	2875	2876	1	1	0

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Example

$$A = \begin{bmatrix} -1.6747 & -0.4747 & -0.1930 & -0.0423 & -0.3439 \\ -0.4747 & -0.0990 & 1.5974 & -0.1492 & -0.2069 \\ -0.1930 & 1.5974 & -0.3087 & -0.1067 & 0.0434 \\ -0.0423 & -0.1492 & -0.1067 & -1.8911 & 0.5842 \\ -0.3439 & -0.2069 & 0.0434 & 0.5842 & 1.3735 \end{bmatrix}, B = \begin{bmatrix} 0.4497 & 0.2410 \\ 0.8504 & -0.5313 \\ 0.6386 & -0.3696 \\ -0.0176 & 2.6961 \\ -0.1881 & -1.1107 \end{bmatrix}.$$

No more than two successive dropouts are allowed.
Actual minimum eigenvalue of the Gramian $= 0.0088$, heuristic value $= 0.4332$.
Corresponding signals:
Actual: $\{0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0\}$,
Approximate: $\{0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0\}$.

Theorem

Suppose (1) is controllable and A has at least one eigenvalue inside the unit circle.

1. *For any two time instants* $t_1 \leq t_2$, $\lambda(t_1) \leq \lambda(t_2)$. *Furthermore,* $\underline{\lambda}(t) = \lambda(t) \leq \lambda$ *and* $\lim_{t\to\infty}\underline{\lambda}(t) = \lambda$.
2. *Suppose*

$$V^{-1}AV = \bar{A} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \bar{B} = V^{-1}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$\bar{W}_{\sigma^*(t)} = V^{-1}W_{\sigma^*(t)}V^{-\mathsf{T}} \text{ and } \hat{W}_{\sigma^*(t)} = \begin{bmatrix} I & 0 \end{bmatrix}^{\mathsf{T}} \bar{W}_{\sigma^*(t)} \begin{bmatrix} I \\ 0 \end{bmatrix}$$

where A_1 *has all eigenvalue strictly inside the unit circle and* A_2 *has eigenvalues on or outside the unit circle. Then,*

$$\lambda \leq \hat{\lambda}(t) \tag{16}$$

$$\text{where} \quad \hat{\lambda}(t) = \|V\|_2^2 \lambda_{min}(\hat{W}_{\sigma^*(t)} - W_t(A_1, B_1) + W(A_1, B_1)). \tag{17}$$

Corollary

Consider a system of the form (1) *which is controllable and stable. Suppose*

$$\bar{\lambda}(t) = \lambda_{min}(W_{\bar{\sigma}}). \quad (18)$$

Then,

1. $\bar{\lambda}(t) \geq \lambda$.
2. $lim_{t\to\infty}\bar{\lambda}(t) = \lambda$.

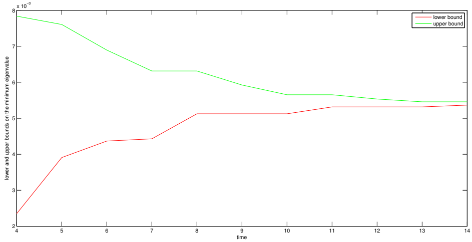

Figure : *A* stable, Automaton: no more than two consecutive dropouts

Conclusion and future work

- Given a finite amount of energy/average energy, we show that the reachability problem is decidable.

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Conclusion and future work

- Given a finite amount of energy/average energy, we show that the reachability problem is decidable.
- We used a partial order on switching signals to reduce computations.
- We provide a heuristic for the same which reduces the amount of computations further (polynomial time).
- We want to characterize a trade-off between the control energy and the cost of the communication network.
- We wish to consider more general models for decentralized control where switching signals associated with different inputs are governed by different automata.

References

- R. M. Jungers and W. P. M. H. Heemels, *Controllability of linear systems subject to packet losses*, Proceedings of the 5th IFAC Conference on Analysis and Design of Hybrid Systems, ADHS'15, Georgia, USA, 2015.
- R. M. Jungers, A. Kundu, and W. P. M. H. Heemels, *On Observability in networked control systems with packet losses*, 53rd Annual Allerton Conference on Communication, Control, and Computing, 2015.
- R. M. Jungers, A. Kundu, and W. P. M. H. Heemels, *Observability and controllability analysis of linear systems subject to data losses*, Archive, 2016.
- T. H. Summers, F. L. Cortesi and J. Lygeros, *On Submodularity and Controllability in Complex Dynamical Networks*, IEEE Transactions on Control of Network Systems, vol. 3, no. 1, pp. 91-101, 2016.
- F. Pasqualetti, S. Zampieri, F. Bullo, *Controllability Metrics, Limitations, and Algorithms for Complex Networks*, IEEE Transactions on Control of Network Systems, vol. 1, no. 1, 2014.
- A. Olshevsky, Eigenvalue clustering, control energy, and logarithmic capacity, System & Control Letters ,Vol.96, pp. 45-50, 2016.
- A. Bemporad, M. Heemels, and M. Johansson, eds., Networked control systems, vol. 406 of Lecture Notes in Control and Information Sciences, Springer-Verlag, Berlin, 2010.
- M. Philippe, R. Essick, G. Dullerud, and R. M. Jungers, Stability of discrete-time switching systems with constrained switching sequences, Automatica, 72, pp. 242-250, 2015.

THANK YOU.