Copyful Streaming String Transducers

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Automata based formal methods

Automata-based approaches:

- model checking
- controller synthesis
- performance evaluation
- model optimization

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Lift this study to transducers:

- language and speech processing
- model-checking infinite state-space systems
- reactive systems
- verification of web sanitizers

Overview

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Streaming String Transducers [Alur, Cerny, 2010]

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Registers: output words

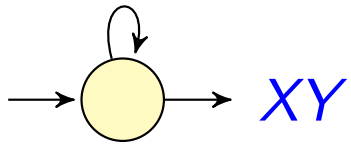
$$w \mapsto w \,.\, mirror(w)$$

Register updates:

- $X := u \cdot Y \cdot v$
- $X := YZ$

X, Y, Z: registers
u, v: words in Σ^*

$$\sigma,\ upd_\sigma : \begin{cases} X := X.\sigma \\ Y := \sigma\, Y \end{cases}$$



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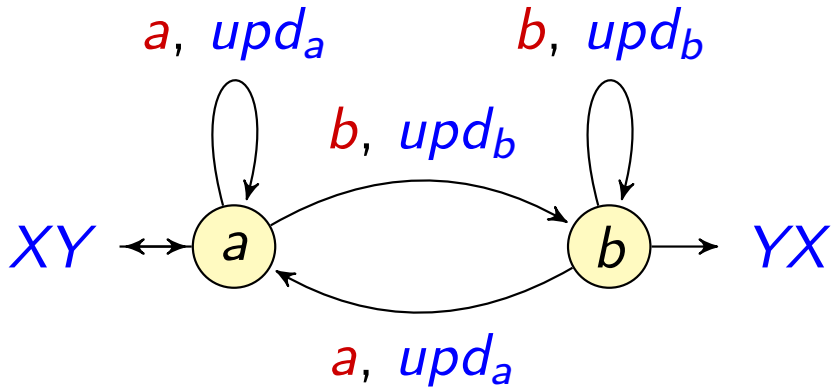

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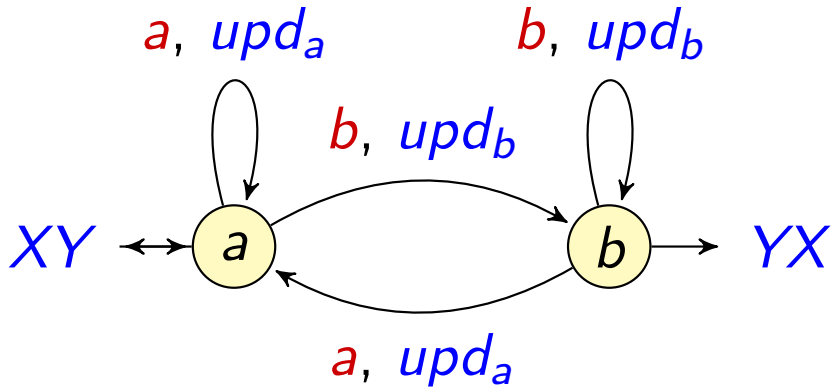
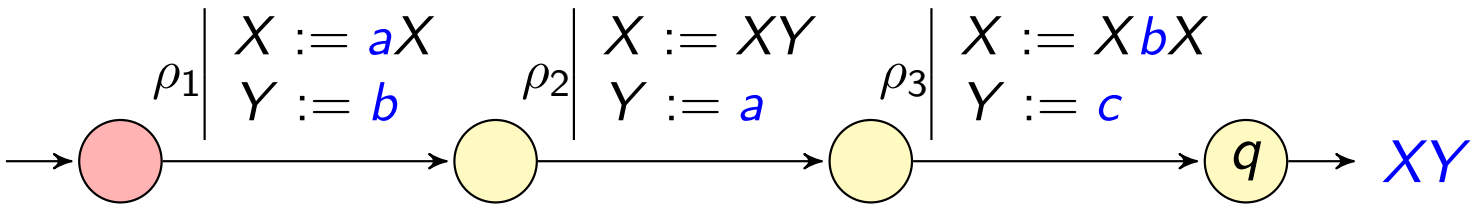


Copyless restriction: a register cannot be copied

$$\text{OK} \begin{cases} X := XY \\ Y := aZb \\ Z := bb \end{cases} \qquad \text{KO} \begin{cases} X := Ya \\ Y := X \\ Z := bY \end{cases} \qquad \text{KO} \begin{cases} X := YaY \\ Y := XZ \\ Z := bb \end{cases}$$

Semantics of SST: two views



Forward interpretation: register valuations

X: ε

Y: ε

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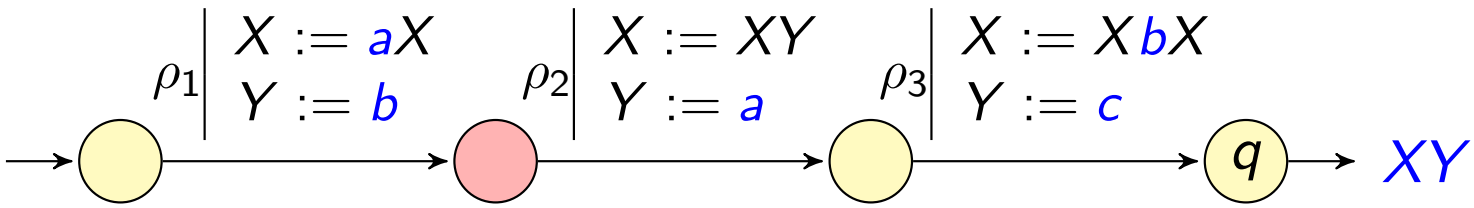

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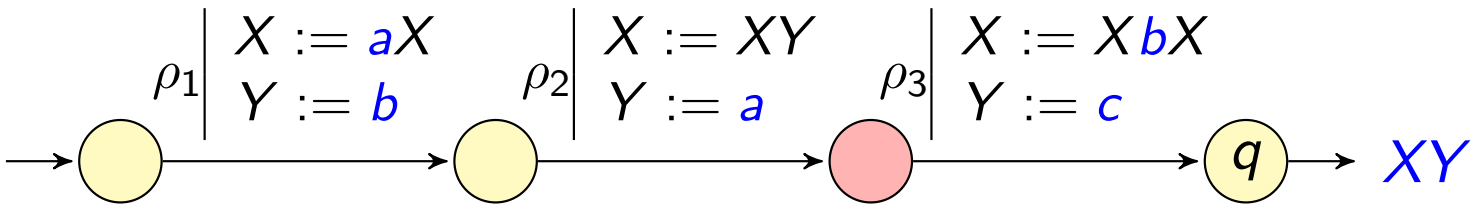

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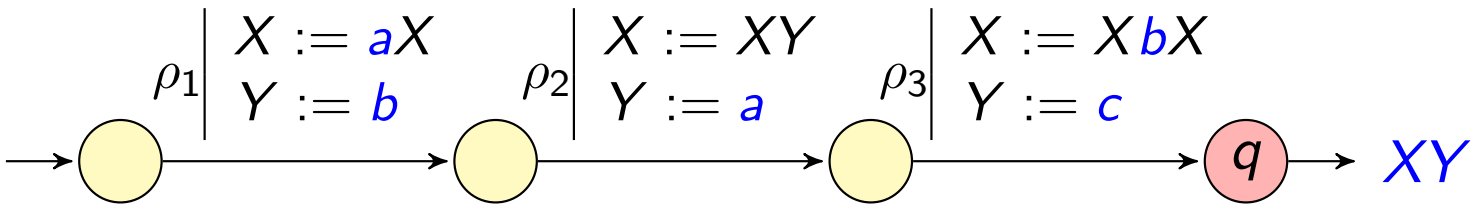

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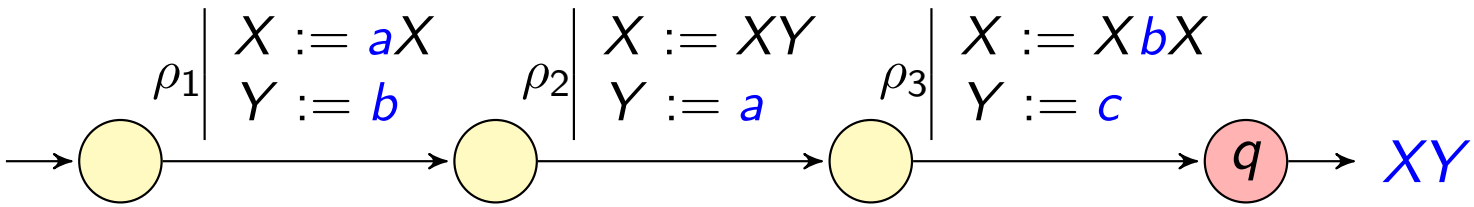

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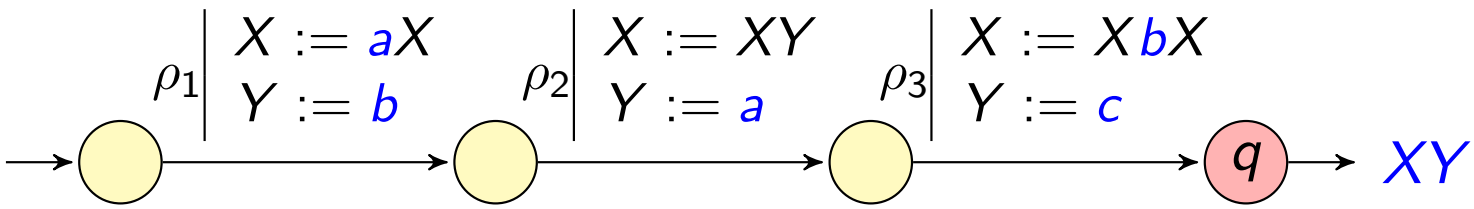

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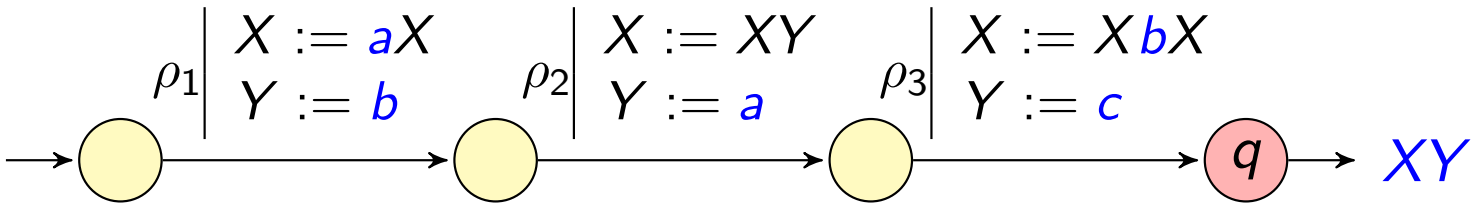

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XY

$\rho_f(q)$

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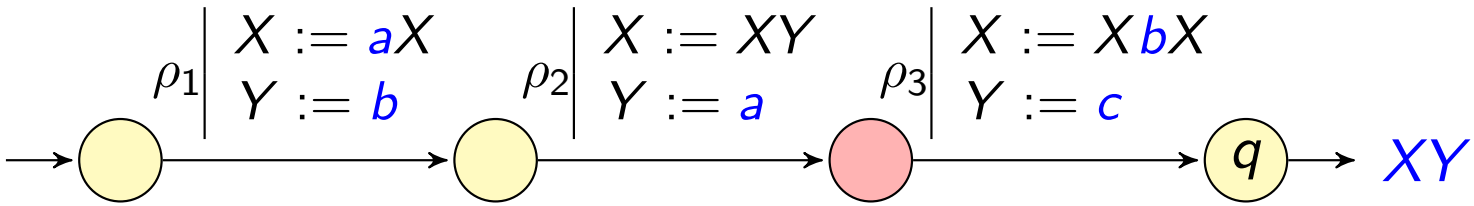

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XbXc	XY
$\rho_3(\rho_f(q))$	

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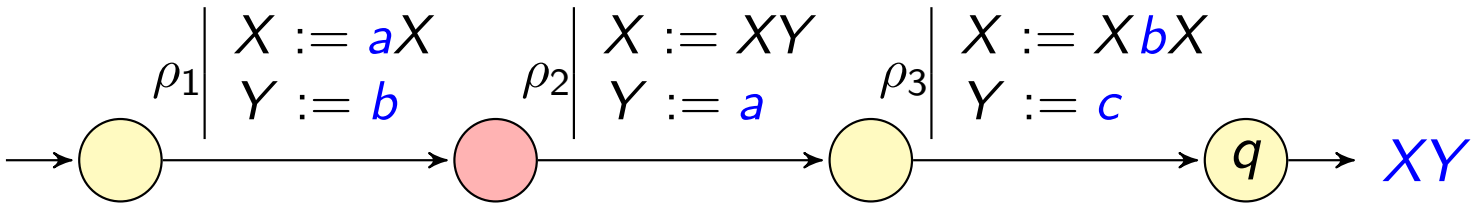

Forward interpretation: register valuations

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Backward interpretation: word over registers

XYbXYc XbXc XY

$\rho_2(\rho_3(\rho_f(q)))$

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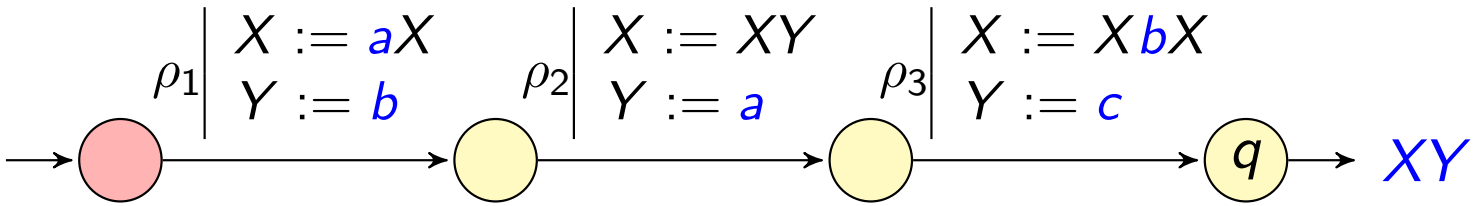

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Backward interpretation: word over registers

aXbbaXbc XYbXYc XbXc XY

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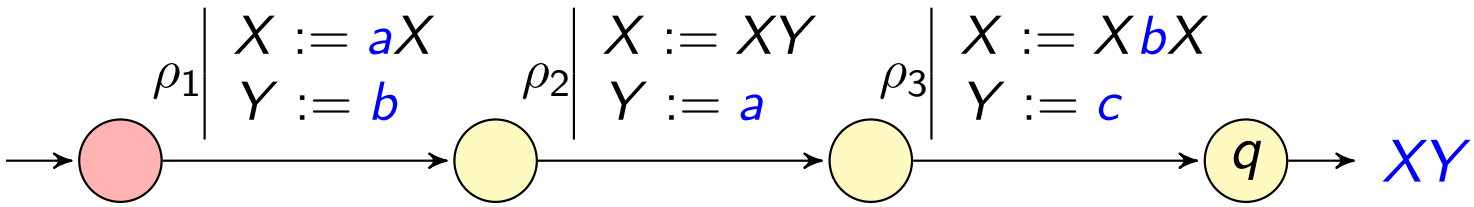

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➜ composition of homomorphisms

Why are SST worth being studied?

"Regular" word-to-word functions:

- copyless SST (and also 1-bounded SST, k-bounded SST)
- deterministic two-way transducers
- MSO-definable word transducers

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Positive results:

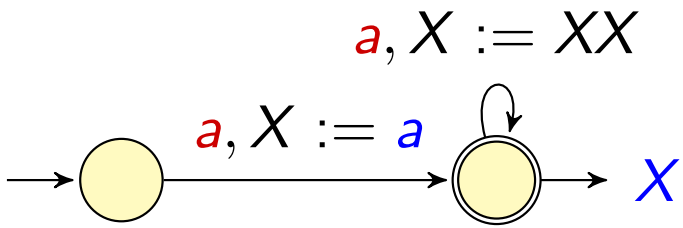
- equivalence of copyless SST is in PSPACE
- functionality of non-deterministic SST is in PSPACE
- type checking is in PSPACE (Given A, B and T, does $T(A) \subseteq B$?)
- closed under composition

What about copyful SST?

Non-linear updates

➜ not anymore linear-size increase

➜ strictly increases the expressive power



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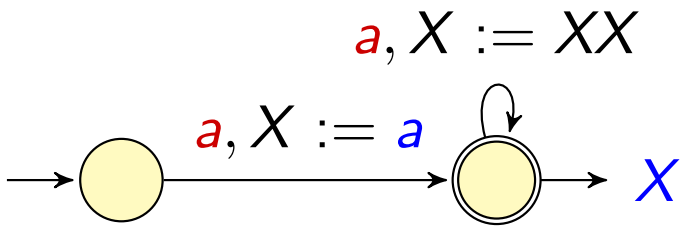

A second example:

$$u\#v \mapsto v[a \leftarrow u]$$

(quadratic blow-up)

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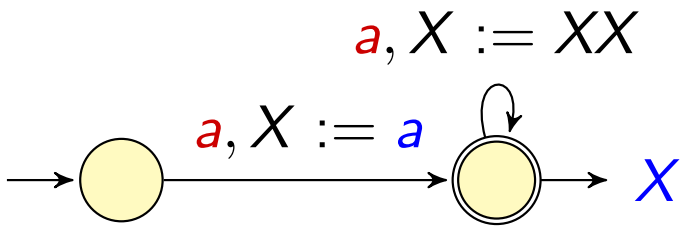

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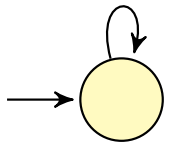

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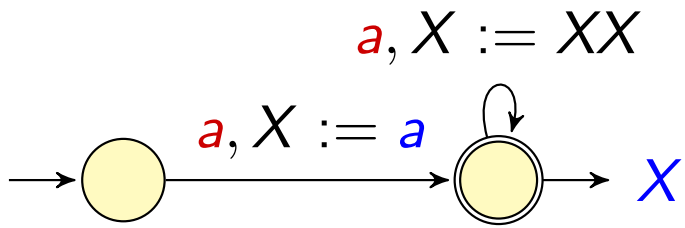

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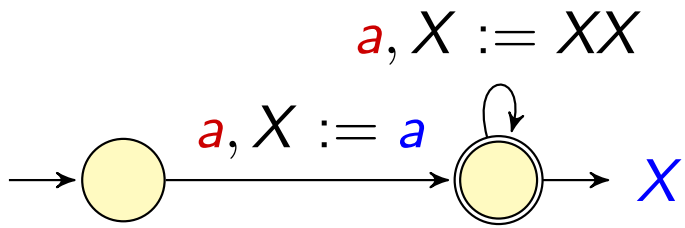

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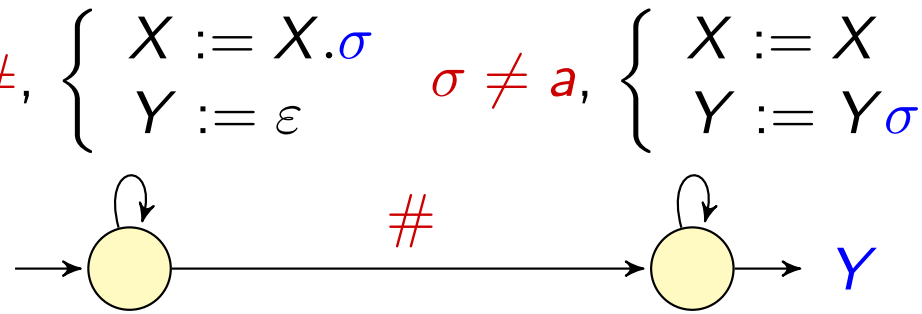

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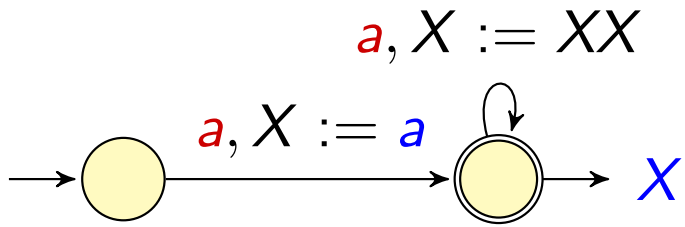

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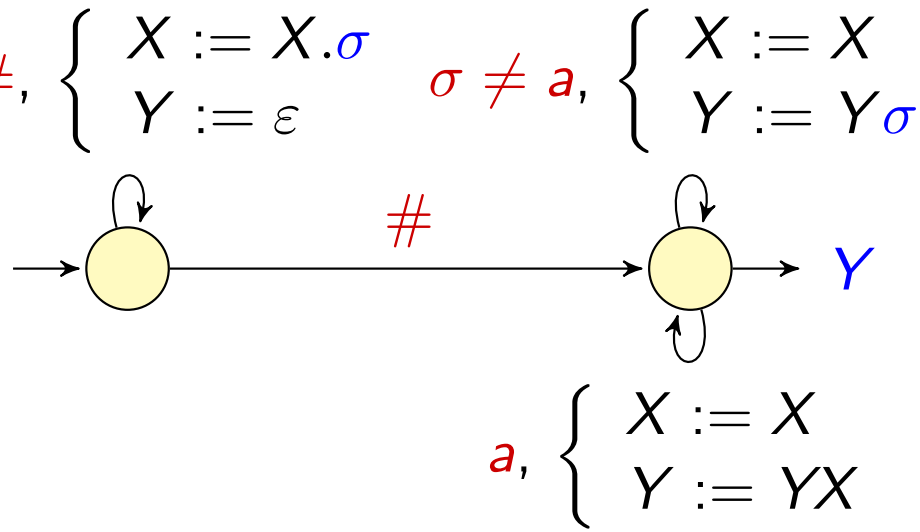

Our results for copyful SST

We obtain decidability results for copyful SST:

- equivalence
- functionality
- copyless-definability: given an SST, does there exist an equivalent copyless one?

Note: decidability of SST equivalence is also a consequence of:

- decidability of the equivalence of top-down tree-to-string transducers [STOC'15]
- recent result on polynomial automata [LICS'17]

Overview

HDT0L systems

An HDT0L system is defined by:

- three alphabets Σ (input), A (working) and Γ (output)
- an initial word $v \in A^*$
- for each $\sigma \in \Sigma$, a homomorphism $h_\sigma : A^* \to A^*$
- a final morphism $h : A^* \to \Gamma^*$

Behaviour:

$$v \qquad \in A^*$$

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Behaviour:

$$\sigma_1 \dots \sigma_k \in \Sigma^* \qquad h_{\sigma_1} \dots h_{\sigma_k}(v) \qquad \in A^*$$

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Theorem (Culik II & Karhumaki'86, Ruhonen'86, Honkala'00)

Given two HDT0L systems H_1 and H_2, one can decide whether $[\![H_1]\!](w) = [\![H_2]\!](w)$ for every $w \in \Sigma^*$.

HDT0L systems: an example

$\Sigma = \Gamma = \{a, b\}$

$A = \Sigma \cup \{\$_1, \$_2\}$

$v = \$_1\$_2$

h_a : leaves Σ unchanged, $\$_1 \mapsto \$_1 a$, $\$_2 \mapsto a\$_2$

h_b : similar to h_a

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Computation of $[\![H_0]\!](baa)$:

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$$baaaab$$

Transformation $w \mapsto w \,.\, mirror(w)$

HDT0L and SST

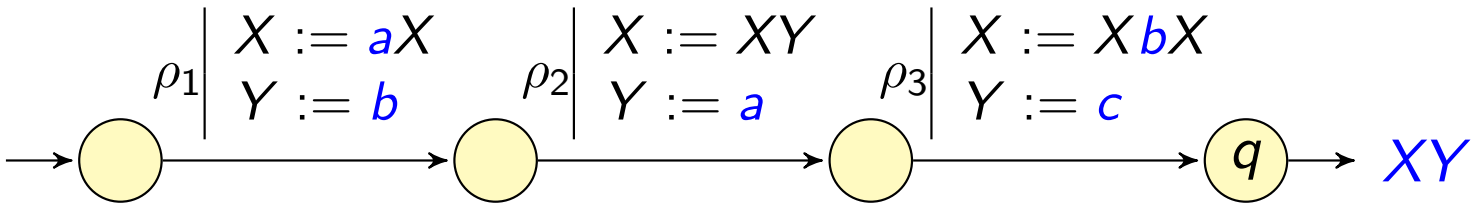
Theorem

HDT0L systems $\equiv$ *total copyful SST.*

Constructions are effective in both directions, in linear-time.

HDT0L and SST

HDT0L $\rightsquigarrow$ total SST:



Backward interpretation: word over registers

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HDT0L and SST

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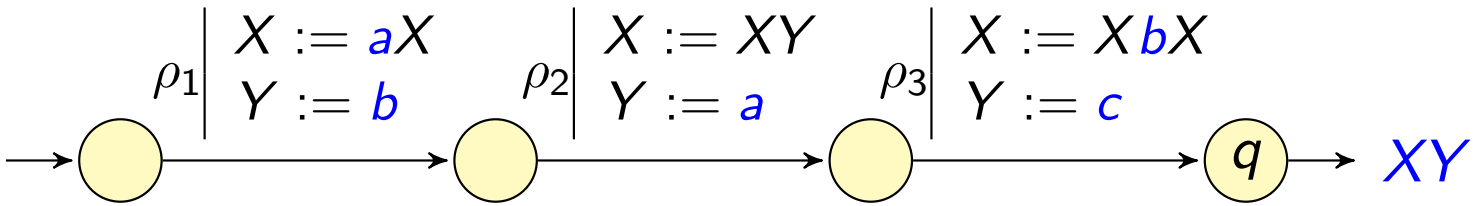

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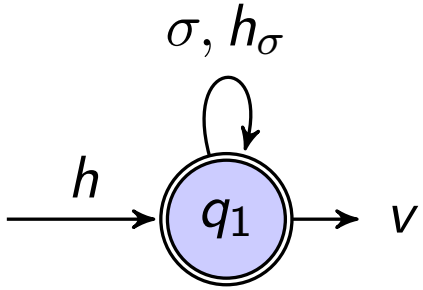
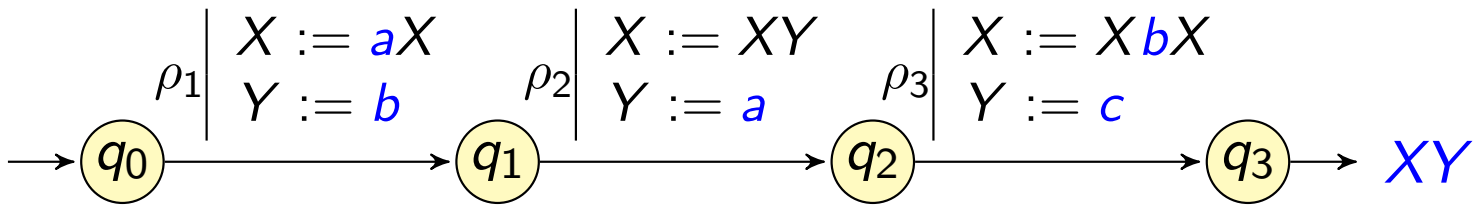
➜ HDT0L are single-state SST!



HDT0L and SST (2)

total SST $\rightsquigarrow$ HDT0L:



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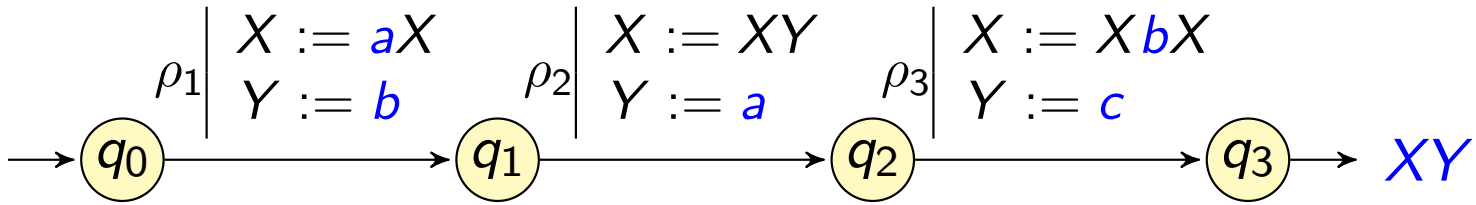


Backward interpretation: word over registers

abbabc aXbbaXbc XYbXYc $X_{q_2} b_{q_2} X_{q_2} c_{q_2}$ $X_{q_3} Y_{q_3}$

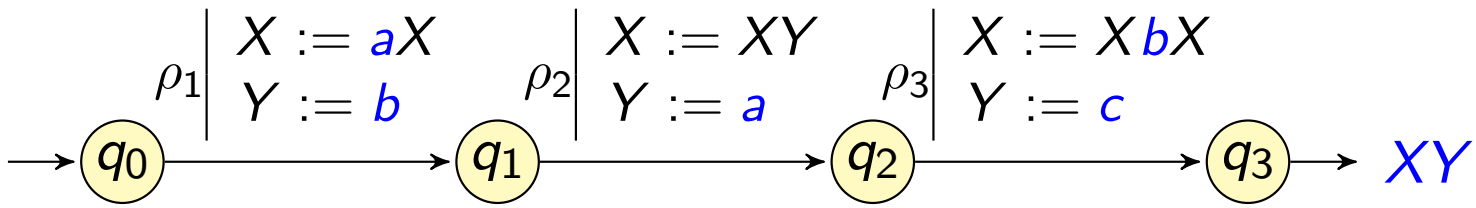
$\rho_{ini}(\rho_1(\rho_2(\rho_3(\rho_f(q_3)))))$

Problem1: states

➜ annotate registers and elements of Σ with Q

HDT0L and SST (2)

total SST $\rightsquigarrow$ HDT0L:



Backward interpretation: word over registers

abbabc aXbbaXbc XYbXYc $X_{q_2} b_{q_2} X_{q_2} c_{q_2}$ $X_{q_3} Y_{q_3}$

$\rho_{ini}(\rho_1(\rho_2(\rho_3(\rho_f(q_3)))))$

Problem1: states

➜ annotate registers and elements of Σ with Q

Problem2: SST are not co-deterministic ($q_1 \xrightarrow{a,\rho_1} q$ and $q_2 \xrightarrow{a,\rho_2} q$)

➜ apply both updates!

$\forall w, \forall q$, there is **exactly one** accepting run on w from q

Final homomorphism h erases useless computations

Consequences

Theorem

Equivalence of copyful SST is decidable, with same complexity as equivalence of HDT0L systems.

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For non-deterministic SST, we can study the functionality problem:
Is it the case that for any two runs on the same input word, the output words are equal?

Using a squaring construction, we reduce it to equivalence:

Theorem

Functionality of non-deterministic copyful SST is decidable.

Overview

From copyful to copyless SST

Exisiting result: Bounded copy SST $\equiv$ copyless SST

A pair (p, X) is problematic iff

1. unbounded register content
2. unbounded copy

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Condition 1.: fixpoint computation on sizes
Condition 2.: reduction to a boundedness problem on products of matrices [Mandel & Simon'77]
➜ Decidability in polynomial time!

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Theorem

Given a copyful SST, one can decide in PTime whether there exists an equivalent copyless SST.

Overview

Summary and Perspectives

- strong link between copyful SST and HDT0L systems:

  copyful SST and HDT0L systems are the same!

- positive results for copyful SST (equivalence, functionality)
- decision of copyless among copyful SST

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Thanks!